Exercise 51

Find the absolute maximum and absolute minimum values of f on the given interval.

$$f(x) = 3x^4 - 4x^3 - 12x^2 + 1, \quad [-2, 3]$$

Solution

Take the derivative of the function.

$$\begin{aligned} f'(x) &= \frac{d}{dx}(3x^4 - 4x^3 - 12x^2 + 1) \\ &= 3(4x^3) - 4(3x^2) - 12(2x) + 1(0) \\ &= 12x^3 - 12x^2 - 24x \end{aligned}$$

Set $f'(x) = 0$ and solve for x.

$$12x^3 - 12x^2 - 24x = 0$$

$$12x(x^2 - x - 2) = 0$$

$$12x(x - 2)(x + 1) = 0$$

$$x = \{-1, 0, 2\}$$

$x = -1$ and $x = 0$ and $x = 2$ are within $[-2, 3]$, so evaluate f at these values.

$$f(-1) = 3(-1)^4 - 4(-1)^3 - 12(-1)^2 + 1 = -4$$

$$f(0) = 3(0)^4 - 4(0)^3 - 12(0)^2 + 1 = 1$$

$$f(2) = 3(2)^4 - 4(2)^3 - 12(2)^2 + 1 = -31 \qquad \text{(absolute minimum)}$$

Now evaluate the function at the endpoints of the interval.

$$f(-2) = 3(-2)^4 - 4(-2)^3 - 12(-2)^2 + 1 = 33 \qquad \text{(absolute maximum)}$$

$$f(3) = 3(3)^4 - 4(3)^3 - 12(3)^2 + 1 = 28$$

The smallest and largest of these numbers are the absolute minimum and maximum, respectively, over the interval $[-2, 3]$.

The graph of the function below illustrates these results.